B.Sc. Semester-II Examination, 2023

MATHEMATICS [Honours]

Course ID : 22114 **Course Code : SH/MTH/203/GE-2**

Course Title : Real Analysis

[OLD SYLLABUS]

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

UNIT-I

1. Answer any **five** from the following questions: 2×5=10

   a) Verify Bolzano-Weirstrass' theorem for the set $\{\frac{n}{n+1} : n \in N\}$.

   b) Is boundedness a necessary condition for a set to have a limit point?

   c) Let the set $S = \left\{\frac{2}{x+1} : x \in (-1,1)\right\} \subset \mathbb{R}$.. Find the set S', where S' is the set of all limit point of the set S.

[Turn Over]

   d) Apply Sandwich theorem to determine the limit of the sequence $\left\{\frac{\sin n}{n}\right\}$.

   e) Give examples of a set which is dense in $\mathbb{R}$ and which is nowhere dense in $\mathbb{R}$.

   f) Test the convergence of the series $\sum u_n$, where $u_n = \sqrt{n^4+1} - \sqrt{n^4-1}$.

   g) Show that the series $\sum \frac{(-1)^{n+1}}{3\sqrt{n}}$ is convergent but not absolutely.

   h) Examine if the set S is closed or open, where S= $\{x \in R: \sin x = 0\}$.

UNIT-II

2. Answer any **four** from the following questions: 5×4=20

   a) i) Show that the integers set $\mathbb{Z}$ has no limit point.

      ii) Examine if the set $S = \bigcup_1^\infty I_n$ is closed in $\mathbb{R}$, where $I_n = \left\{x \in \mathbb{R} : \left(\frac{1}{3}\right)^n \le x \le 1\right\}$. 2+3=5

   b) i) Let S be a non-empty subset of $\mathbb{R}$, bounded below and $T = \{-x : x \in S\}$. Prove that the set T is bounded above and $\sup T = -\inf S$.

ii) Verify Bolzano-Weierstrass theorem for the set $\subset \mathbb{R}$, where $S = \left\{\frac{n-1}{n+1} : n \in \mathbb{N}\right\}$. 3+2=5

c) i) Show that every compact subset of $\mathbb{R}$ is closed.

ii) Prove or disprove: The set $S = \{x \in \mathbb{R} : \sin\frac{1}{x} = 0\}$ is compact. 3+2=5

d) Examine the convergence of the series $\frac{1}{1+2^{-1}} + \frac{1}{1+2^{-2}} + \frac{1}{1+2^{-3}} + \cdots$ 5

e) Prove that union of any finite number of closed sets is closed. Does this result hold for union of arbitrary family of closed sets? Justify your answer. 3+2=5

f) If $\Sigma\, {u_n}^2$ and $\Sigma\, {v_n}^2$ be both convergent prove that the series $\Sigma u_n v_n$ is absolutely convergent. 5

UNIT-III

3. Answer any **one** of the following questions: 10×1=10

a) i) If $\{u_n\}$ and $\{v_n\}$ are two real sequences converges to A and B respectively. Then prove that

$$\lim_{n} \frac{1}{n}(u_1 v_n + u_2 v_{n-1} + \cdots + u_n v_1) = AB .$$

ii) Find the limit point of the set S= $\{(-1)^m + \frac{1}{n} : m \in N, n \in N\}$.

iii) Test for convergence of the series $\Sigma \frac{1}{\sqrt{n}} \tan\frac{1}{n}$. 5+3+2=10

b) i) Show that union of two enumerable sets is enumerable.

ii) Show that the intersection of an arbitrary collection of closed sets in $\mathbb{R}$ is a closed set.

iii) Let $\sum u_n$ be a convergent series of positive real numbers, prove that $\sum \frac{u_n}{n}$ is convergent.

iv) State Leibnitz's Test for alternating series. Apply Leibnitz's test to show that the series $\frac{1}{1+a^2} - \frac{1}{2+a^2} + \frac{1}{3+a^2} - \cdots$ is convergent.

2+2+2+(1+3)=10

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